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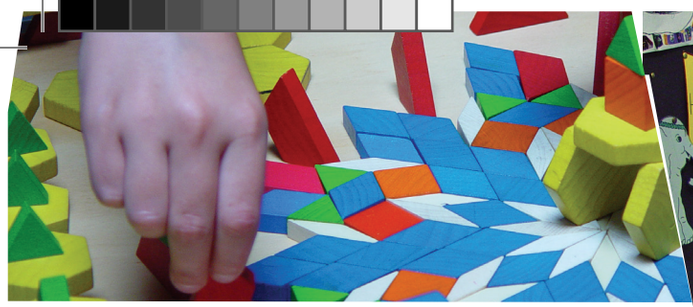


Foreword

For many decades, the use of manipulative materials to assist young children in their learning of mathematics has been recommended. The advocacy of great educators such as Maria Montessori, Zoltan Dienes and Catherine Stern encourages a wide acceptance of the use of manipulative materials, especially in primary school classrooms. Once, it was felt that simply giving students manipulatives to use in mathematics lessons would be enough to develop an understanding of mathematical concepts. This is not true. Manipulatives in and of themselves do not teach—skilled teachers do.

This series—*Hands-on mathematics*—is designed to help teachers who are trying to make the most of students' experiences with manipulatives. We believe it is better to use a few well-chosen products rather than an array of 'bits and pieces'. We recommend 'a lot of a little' rather than 'a little of a lot' when it comes to working with manipulatives. It is better to focus on a few well-chosen manipulative materials so that students will have an adequate supply of pieces. Nothing is more frustrating than not having enough to finish 'creating a design' or 'building that masterpiece'. As well, it is important that sufficient materials are available to allow models to be left on display in the classroom.

Frequently, when we work with students and teachers in classrooms and workshops, a range of common concerns is raised. Let us share a few with you.

Why use manipulatives?

When used as part of a well thought-out lesson, manipulatives can help students 'come to grips' with difficult concepts. The key to good use of manipulatives is for teachers to have a clear goal in mind when using them. This will help maintain the intention of the lesson and focus responses to any questions asked during the lesson. Teachers will have a clear idea of what to look for when observing students using manipulatives.

As Richard Skemp, the famous educational psychologist said, 'It is as though their thinking was out there on the table'.

We have observed how students experiment with ideas willingly. If, at first, satisfaction with an idea is not achieved, students will seek another solution. We do not see this happening as frequently when students are expected to work with abstract statements such as equations and written problems.

The skilled use of manipulatives—note, we said the *skilled* use of manipulatives—will enhance mathematics outcomes. Poor use may be detrimental to student attainment. This series of books is designed to ensure skilled use of manipulatives in the classroom.

Is there a difference between a mathematics manipulative and a mathematics teaching aid?

We believe there is a big difference between the two types of materials.

A child can interact, even take control of a good mathematical manipulative; whereas a teaching aid tends to control the learning experience. Too often, a teaching aid is used as a 'telling' support rather than a learning support and experience has taught us that 'telling' is not a very successful method of teaching mathematical ideas.

How will I know whether the students are learning anything?

Observe the students as they work with the manipulatives. Don't worry if they solve a problem in a way different from what you expected. Ask questions. Encourage students to explain their thoughts or write about their experience.

In fact, actively engage with the students as their thought processes emerge. Simply using manipulatives is not enough. Students need to be given time to reflect on their activity and share their thoughts with a group or the whole class. The teacher plays a vital role in helping students connect new knowledge with old. Language plays a key role throughout this learning process.

What evidence can I show that students are learning or have learnt…?

Some teachers are concerned about the lack of written evidence to substantiate learning when manipulatives form a large part of the lesson. There are several ways a student might record his/her findings:

- *writing about the experience*
- *sketching or drawing any models produced*
- *photographing any models produced*
- *presenting 'learning tours' to students in other classrooms*
- *maintaining a learning journey logbook.*

Actually, when preparing this type of learning evidence, students have a wonderful opportunity to reinforce their own learning.

How do I manage the use of manipulatives?

Some teachers worry that students will only play with the manipulatives and not pay attention, or worse still begin to throw the material around. These are genuine fears which will decrease as experience, both by the students and teacher, increases. As with any 'new toy' there will be a 'novelty effect'. The first time you introduce a manipulative, allow time for the students to explore.

Set some simple rules and limits for the way the material is used and enforce these early on. Students will soon learn to respect the material.

Throughout this book, management ideas are presented. We encourage you to adopt them as your own.

Picture puzzles

D T E S

Purpose

To match blocks to figures (congruency).

To cover figures with a given set of blocks.

Using pattern blocks to build shape recognition skills

Create a design to fit onto a single sheet of paper. We have created a design to illustrate each activity.

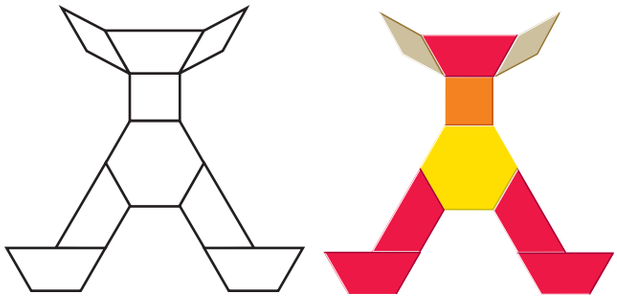
The students will require some blank white paper and some pencils. Ask the students to create a design that fits onto a single blank page.

Depending on the age and ability of the students, several options exist.

Encourage students to create a set of similar challenges.

Option 1 is the simplest as it simply involves students in matching individual pattern blocks to fit into the design.

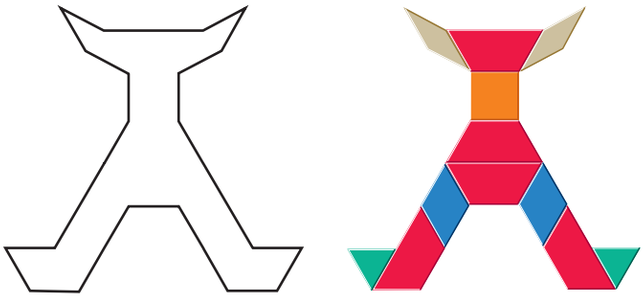
1



Trace around each individual shape in the design.

Option 2 is more sophisticated as different pattern blocks may be used to complete the design. For example, instead of using a single yellow hexagon a student might use two red trapeziums or six green triangles to cover the same area. Challenges such as 'fill the design using the least/most blocks' can be set.

2

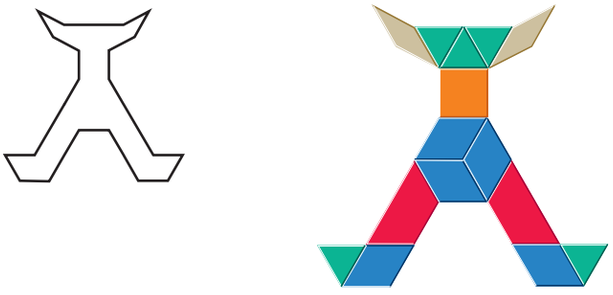


Trace around the outside of the design.

Option 3 is even more difficult as the size of the design and the Pattern Blocks do not match. The original drawing and the final version will be the same shape (not the same size) but not congruent (same shape and size).

Some teachers prefer to introduce an 'in-between' stage by indicating the shapes in the reduced model.

3



Trace around the outside of the design and reduce or enlarge the design on the photocopier. Then recreate the original design.

D T E S

Exploring angles

Purpose

Using prior knowledge such as a right angle is 90º or a full circle is 360º to determine the angle sizes of each of the pattern blocks.

Angle facts

An angle is the space between two straight lines which meet at a common point. The point may be called a vertex.

An angle is measured by the amount of turn, using degrees, which are indicated by º (finer measurement uses the terms minutes and seconds).

Types of angles

acute angle – less than 90º

right angle – 90º

(Note the special sign ⌝ used to indicate a right angle.)

We have met students searching for 'left' angles!

obtuse angle – more than 90º but less than 180º

straight line – 180º

reflex angle – more than 180º but less than 360º

circle – there are 360º in a circle; some may say a revolution

Polygons

Polygons have a definite number of degrees of their internal angle.

triangle: 180º

quadrilateral: 360º

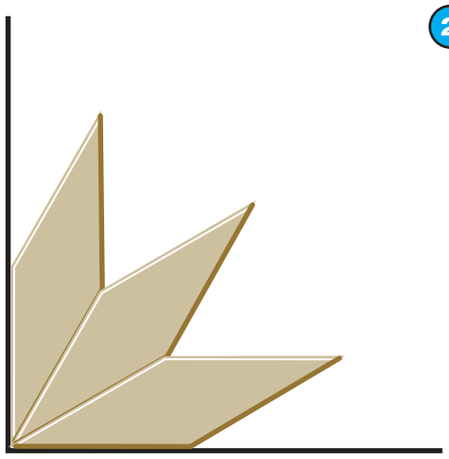
hexagon: 720º

(Note: Each angle in a regular hexagon is 120º)

Students will discover that all the angles of a regular polygon are the same. Not all polygons are regular—consider the regular hexagon in the set of pattern blocks with other hexagons.

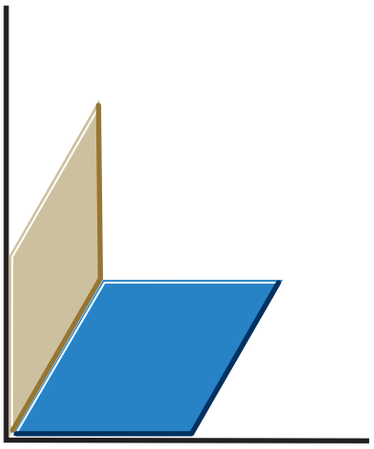
An introduction to angle

1



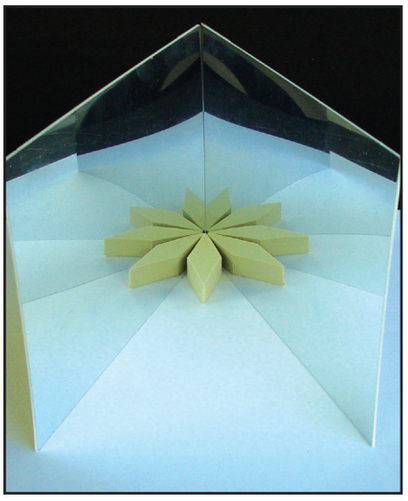
I know a corner of a square is a right angle. What is the small angle of the tan rhombus?

2



Now I know the smaller angle in the tan rhombus, what is the small angle in the blue rhombus?

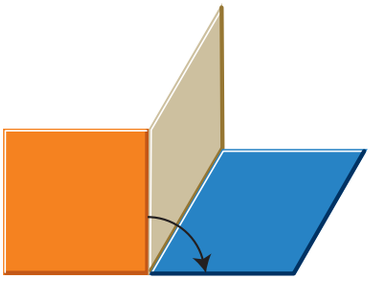
3



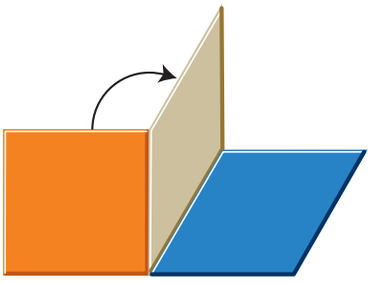
Use the tan rhombus and the hinged mirror to create a picture of twelve tan rhombuses.

We know that the small angle of the tan rhombus is 30º. In the mirror we made a circle; how many degrees altogether?

4 Combine shapes to make different angles.



How many degrees in the angle above?



Use your knowledge to calculate this angle.

5 An assessment idea: Using the overhead projector and overhead projector pattern blocks, students explain their method for finding angles.

D T E S

Exploring hexagons

Purpose

To encourage students to use their spatial skills (reflection, rotation) to work mathematically to create hexagons.

Watch your language

Throughout our teaching relationships with students, a major problem frequently occurs: the meaning of words. Here is one which needs careful consideration:

regular –

regular fries

regular (meaning punctual)

regular army

regular (meaning periodic, rhythm, seasonal)

regular (as in health measurement)

I am sure you can add to this list. Now consider entries in a dictionary:

regular: usual, conforming, orderly, even, steady, happening at fixed times, following a rule or procedure …

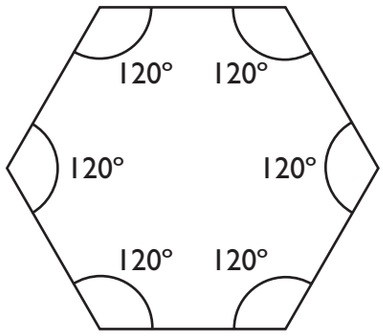
There are fourteen meanings of regular in my 'essential' dictionary.

So how does a student feel when we refer to this shape  as a regular hexagon? And then another regular shape  as an equilateral triangle and  as a square, rather than a regular quadrilateral?

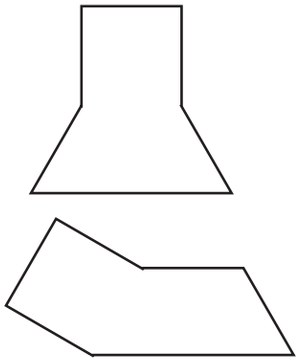
Simply, learning the vocabulary is possibly as difficult as coming to grips with the mathematics in and of those shapes.

Not all polygons are regular

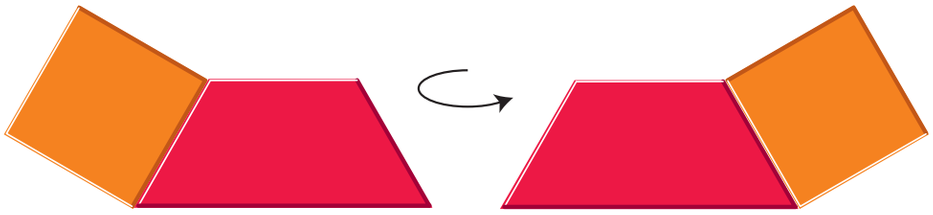
1 This is a regular hexagon. All the sides are the same length and all the angles are the same size.



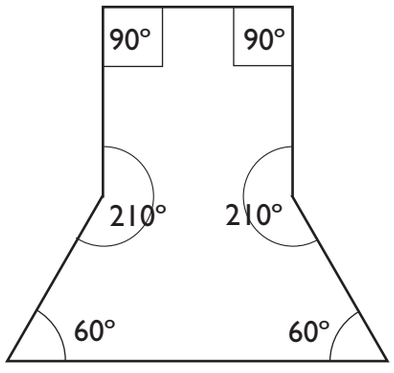
2 These shapes are hexagons. Each shape has six sides.



3 These are the same shapes. One hexagon may be reflected (flipped over) to fit exactly on the other one.



A hexagon has six sides and the sum of all the angles is 720°.



D T E S

Purpose

Strengthening arithmetic skills.

Shopping with pattern blocks

Strengthening arithmetic skills

We have discovered that arithmetic skills flow so much more easily when supported by appropriate manipulative material. The visual and/or concrete representations strengthen a student's intuitive skills, a process often ignored as mathematical ideas are developed. I frequently play 'Guess what's in my pocket'. A green triangle has a value of 6. 'In my pocket I have blocks which show half of 72 less six. What blocks might I have?' Most times the challenge is met eagerly.

A few decades ago, these ideas were introduced under the broad heading of value-relations. One of the natural results of this thinking is the appreciation of fraction, ratio and percentage concepts. By rushing into this conceptual area without the preceding experiences, as described on pages 25–26 and 66–67, both students and many teachers become confused.

Consider the mathematical procedures being reinforced and the number skills being developed in these pattern block shopping games. Our experience has shown that students become really involved in these make-believe shopping experiences.

Creative number operation skills

A South Pacific island nation has decided to use shells as their form of currency.

If  = 5 shells (s)

1 Different ways to show 20 shells

with two blockswith three or more blocks

10s + 10s

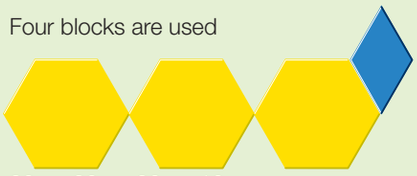
15s + 5s

5s + 5s + 5s + 5s

10s + 5s + 5s

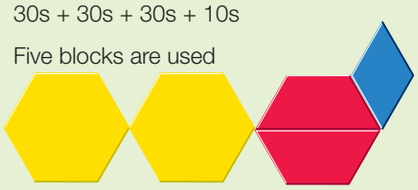
2 The 100 shell collection

Four blocks are used



30s + 30s + 30s + 10s

Five blocks are used

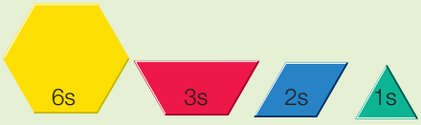


30s + 30s + 15s + 15s + 10s

Make 100 shell models using 4, 5, 6 and 12 blocks in a model.

3

Change the values of the pattern blocks and carry out similar activities to the above: Example:



4 Challenges

- You have 10 hexagons (10H) and you purchase a 3 kg bag of sugar and 2 kg of apples. What change will you receive?
- Detergent is half price today. How much will a bottle cost?
- Purchase three items so that you have almost no change remaining from 10 hexagons (10H).
- Plan a business venture marketing toffee apples. (I kg of apples needs about 2 kg of sugar to make a suitable toffee.) No student may invest more than 2H to finance the business. How many shareholders will be needed?

Guide to using these photocopiable resources

Cut along all dotted lines, fold all solid lines so that each grocery item stands up.

There are two distinct activities on these sheets.

1. Grocery items with illustrated pattern block prices
2. The same grocery items with a symbolic code to indicate the prices. The code will be known because it is introduced in pattern block trading.

- Decide on the value of the triangle before visiting the shop.
- Grocery items may be cut out to make stand-up figures. This will help the students sort the various items.
- Encourage the collection of used grocery containers to make a class shop. Students may translate prices from Shells to pattern block money; for example, an item that costs 210s will cost 3H1Tr (3 hexagons and a trapezium) in pattern block money when a triangle = 10s.
- Challenge students to create a pattern block shop 'special' flyer.
- Invent a simple code to describe the prices.